Computing a Minimum-Cost k-hop Steiner Tree in Tree-Like Metrics

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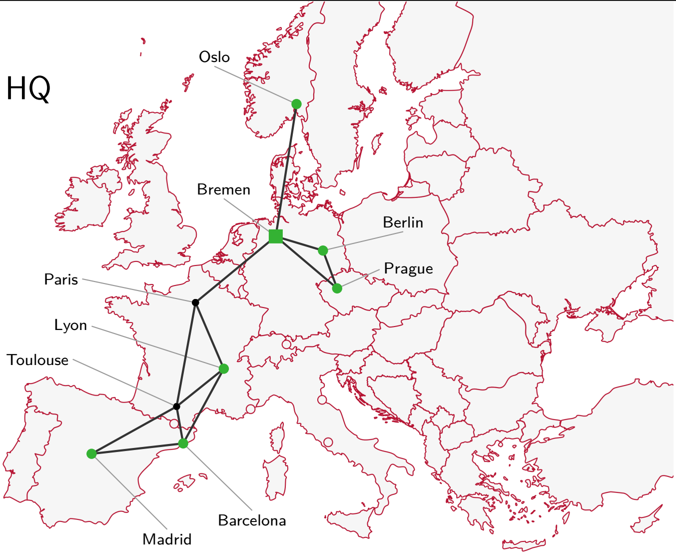
Minimum-Cost *k*-hop Steiner Tree

Transportation network

- Supply customers from company HQ

Minimize transportation costs



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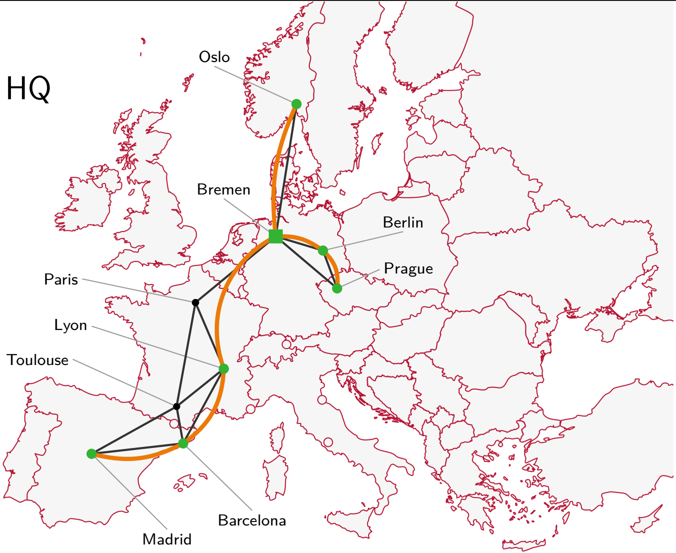

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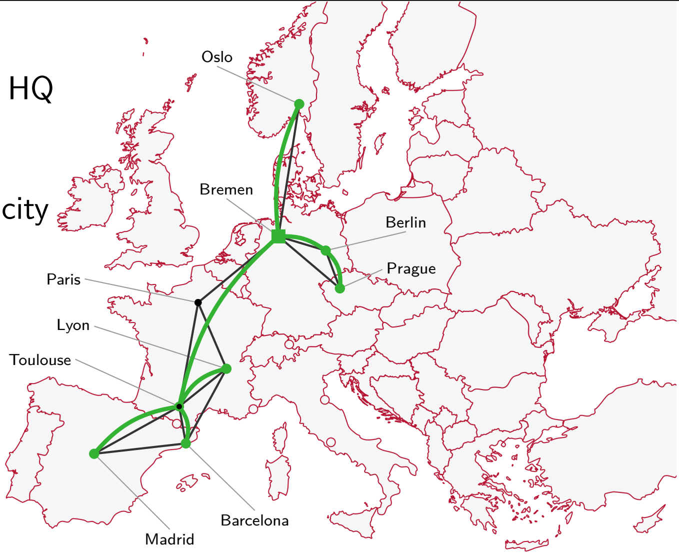
Limit number of stages to reach any city



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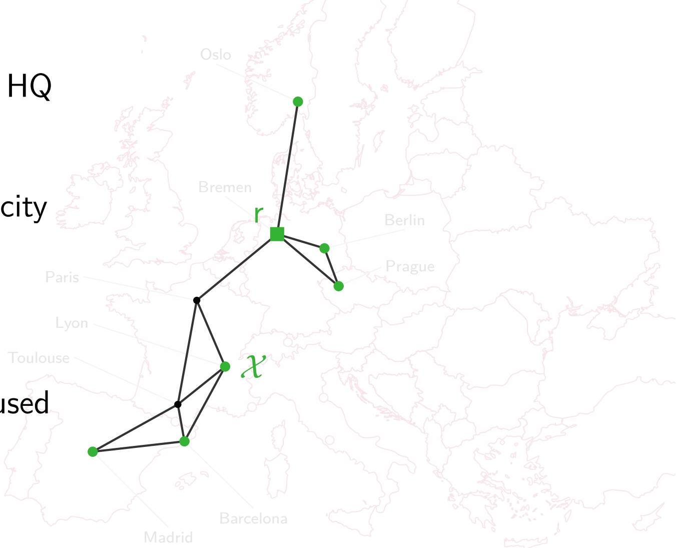
k-hop Steiner Tree: connect terminals $\mathcal{X}$ with a tree rooted at r of depth $\leq k$

Minimize the sum of edge distances used



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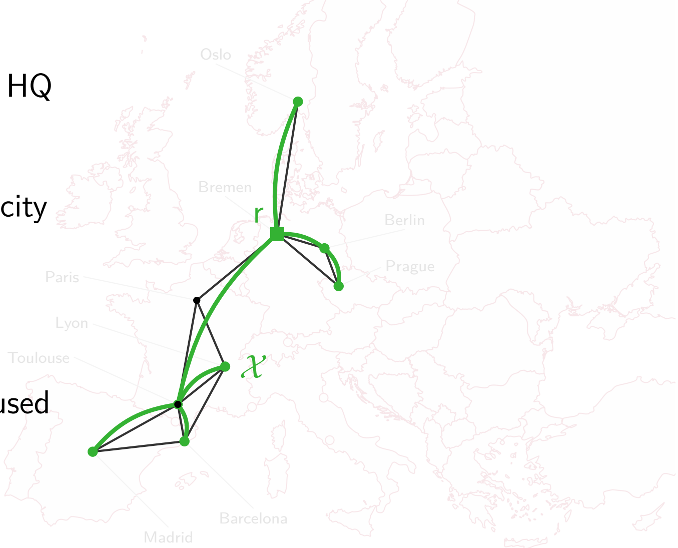

Overview

Best algorithms

- Metric graphs: $O(\log n)$ and $(1.52 \cdot 9^{k-2})$ -apx [Althaus et al. '05 & Kantor, Peleg '09]
- Euclidean space: QPTAS for k constant [Laue, Matijević '08]

Contributions

- Path metric: Exact solution in $O(kn^5)$
- Metric of treewidth ω: Exact solution in $n^{O(k\omega)}$

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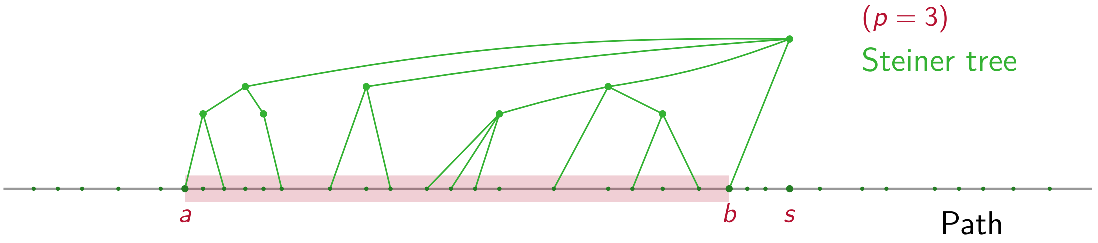
Contributions

- Path metric: Exact solution in $O(kn^5)$
- Metric of treewidth ω: Exact solution in $n^{O(k\omega)}$
- Bounded highway dimension metrics *(Building on [Feldmann, Fung, Könemann, Post 2018])*
  - Solution of cost $(1+\varepsilon)\mathrm{OPT}_{k-1}$ in quasi-polynomial time (QPTAS with $+1$ hop resource augmentation)

Path Metric: Dynamic Program Idea

Can guess a subgraph *below* any node in the k-hop Steiner Tree

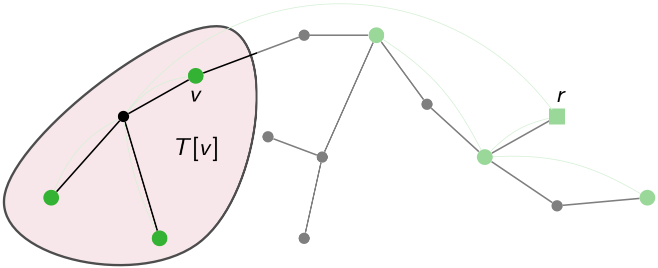
$A[p, s, a, b] :=$ min-cost of p-hop tree
rooted at s
covering $[a, b] \not\ni s$, $\quad p \in [k],\ s, a, b \in [n] = V$



Tree Metric: Dynamic Program Idea

DP cell: subtree $T[v]$ of the metric space rooted at v

Anchoring Guarantees:



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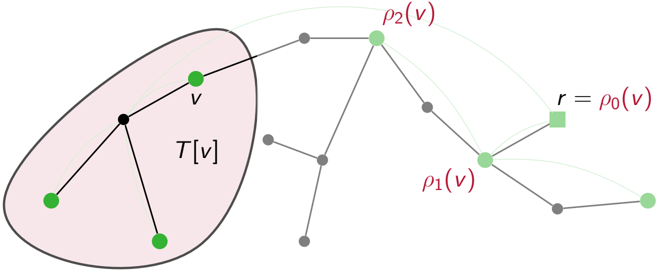

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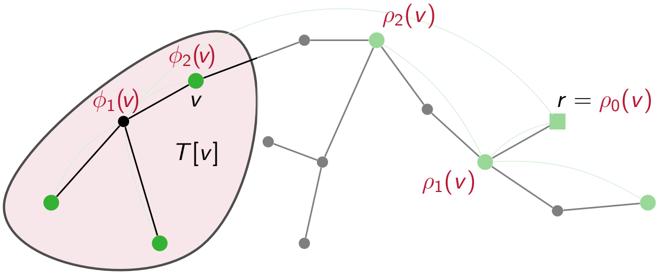

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$$A[v, \rho_1(v), \ldots, \rho_k(v), \phi_1(v), \ldots, \phi_k(v)]$$

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