# Online Scheduling of Task Graphs on Hybrid Platforms

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# Breaking down the title

**Hybrid Platforms**

- Many CPUs + few accelerators (GPUs, Xeon Phis, . . . )

**Task Graphs (DAGs)**

- Used in runtime schedulers (StarPU, OmpSs, XKaapi, . . . )

**Online Scheduling**

- Unknown graph
  - tasks not submitted yet
  - depends on results
- Advantages vs offline
  - quicker decisions
  - robust to inaccuracies
- *Semi-online: partial information, e.g., bottom-levels (≈ critical path)*

**Main challenge: take binary decisions without knowing the future**

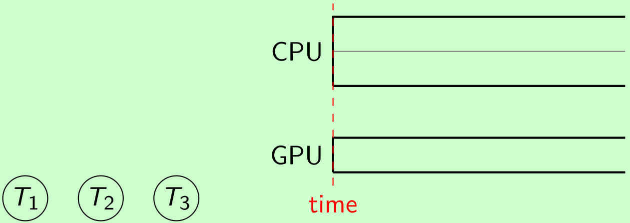
# Model and toy example

## Model

- $m$ CPUs $\geq$ $k$ GPUs
- Graph of tasks $T_i : \left\{ \overline{p_i} = \text{ CPU time ; } \underline{p_i} = \text{ GPU time} \right\}$
- Online: only available tasks are known

## Objective: minimize makespan

**Example (2 CPUs, 1 GPU)**



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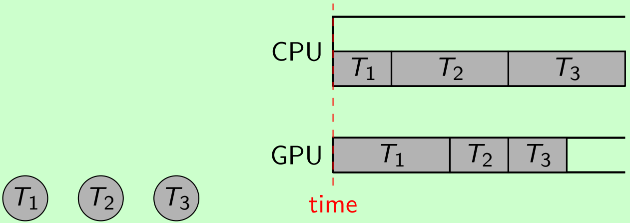

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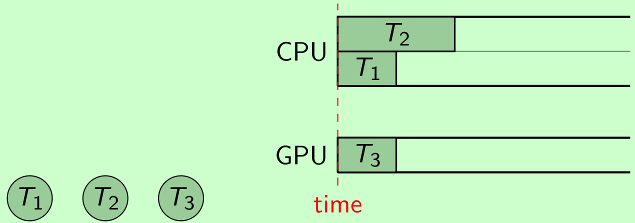

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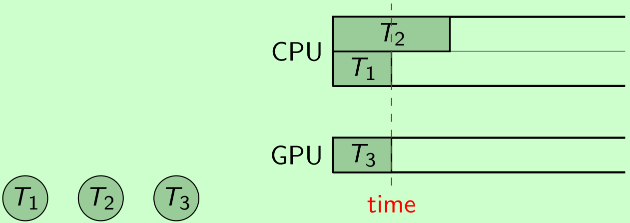

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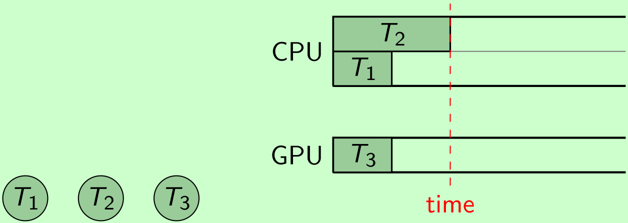

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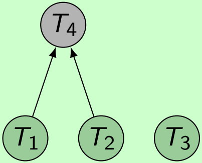


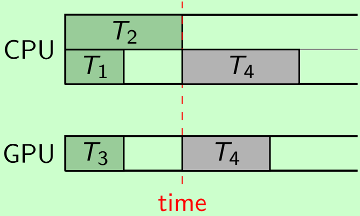

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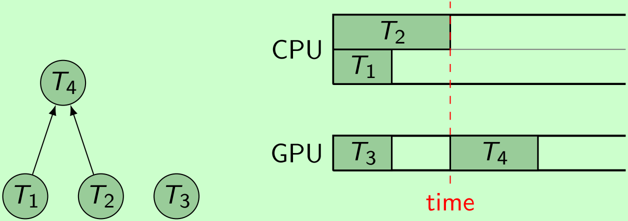


# Related work

$m$ CPUs, $k$ GPUs

## Existing offline algorithms (NP-Complete)

- Independent tasks:
  - $\frac{4}{3} + \frac{1}{3k}$ - approx [Bleuse, Kedad-Sidhoum, Monna, Mounié, Trystram 2015]
    Expensive PTAS [Bonifaci, Wiese 2012]
  - Low-complexity: 2 - approx [Canon, Marchal, Vivien 2017]
    3.41 - approx [Beaumont, Eyraud-Dubois, Kumar 2017]
- DAG: 6 - approx (LP rounding) [Kedad-Sidhoum, Monna, Trystram 2015]

## Existing online algorithms

- Independent tasks: 4 - competitive [Imreh 2003]
  3.85 - competitive [Chen, Ye, Zhang 2014]
- DAG: $4\sqrt{\frac{m}{k}}$ - compet. ER-LS [Amarís, Lucarelli, Mommessin, Trystram 2017]

# Outline

*m* CPUs, *k* GPUs

1. **Lower bounds on online algorithms**
   - No online algorithm can be $< \sqrt{m/k}$ - competitive
2. **Propose improvements of ER-LS**
   - Competitive ratio
   - Average performance
   - Validation on simulations

# Lower bound

$m$ CPUs, $k$ GPUs

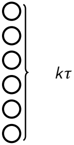
**Theorem**

*No online algorithm $\mathscr{A}$ is $< \sqrt{m/k}$-competitive for any $m$, $k$.*

**Proof (where $\tau = \sqrt{m/k} = 3$):** graph built in $n\tau$ phases.

Phase 1 - $k\tau$ independent tasks $\left\{\overline{p_i} = \tau \ ; \ \underline{p_i} = 1\right\}$: $\mathscr{A}$ needs a time $\tau$

Graph with
$k = 2$, $n = 3$

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Phase 2 - same as phase 1, but are successors of the last task

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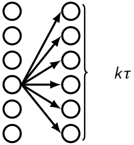


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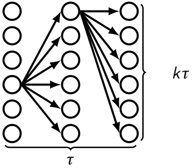


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Phase $x$ - ...

$\Longrightarrow$ Makespan obtained by $\mathscr{A}$: $n\tau^2$

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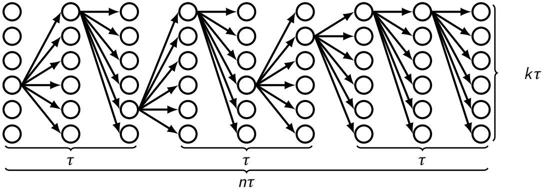


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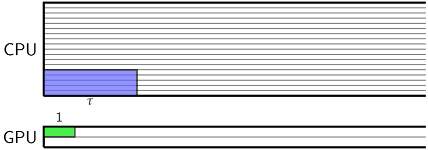


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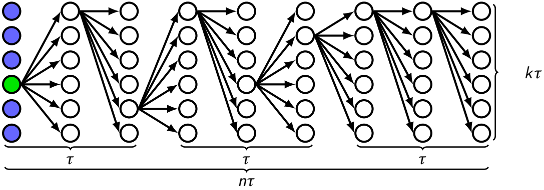

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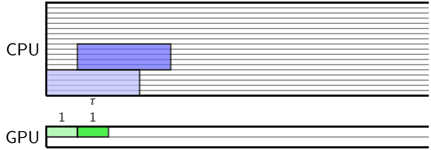


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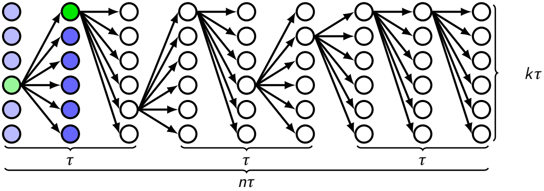


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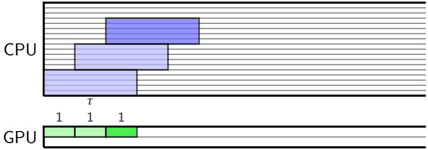
$\tau$ phases

OPT $\Rightarrow 2\tau$

$\mathscr{A} \Rightarrow \tau^2$

Lower bound: $\frac{1}{2}\tau$

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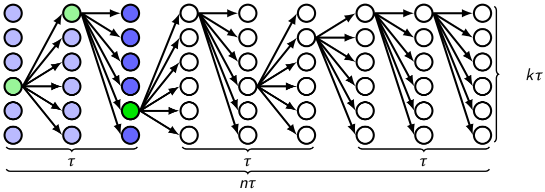


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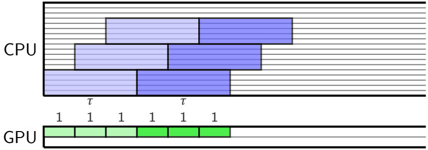
$2\tau$ phases

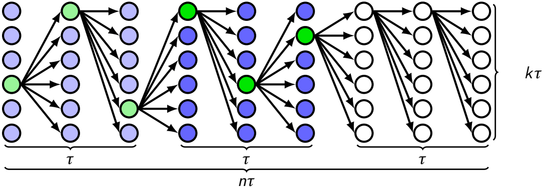
OPT $\Rightarrow 3\tau$

$\mathscr{A} \Rightarrow 2\tau^2$

Lower bound: $\frac{2}{3}\tau$




Graph with
$k = 2,\ n = 3$



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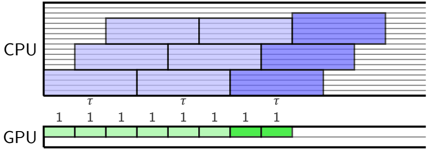
$n\tau$ phases

OPT $\Rightarrow (n+1)\tau$

$\mathscr{A} \Rightarrow n\tau^2$

Lower bound: $\frac{n}{n+1}\tau$

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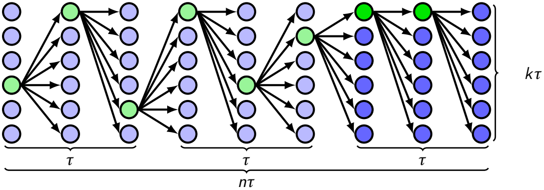


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# Generalized lower bounds

Recall previous lower bound: $\sqrt{m/k}$, for $m$ CPUs, $k$ GPUs

**Precomputed information**

- Bottom-level (≈ remaining critical path) does not help
- All descendants: non-constant LB $= \Omega\left((m/k)^{1/4}\right)$

**Powerful scheduler**

- *Kill* + *migrate* does not help
- *Preempt* + *migrate* hardly helps

**Note: allocation is difficult**

- How to choose which tasks to speed-up?
- Fixed allocation: 3-competitiveness

# ER-LS algorithm ($4\sqrt{m/k}$-competitive, [Amarís et al.])

$m$ CPUs, $k$ GPUs

## Main concept

- Pick any available task $T_i$
- Allocate $T_i$ to CPUs or GPUs
- Schedule it as soon as possible

**Where to allocate an available task $T_i$**

```
If T_i can be executed on GPU before time p̄_i:
  ▸ put T_i on GPU
Otherwise:
  ▸ if p̄_i / p_i ≤ √(m/k): put it on CPU
  ▸ else               : put it on GPU
```

# Our proposition: QA (Quick Allocation) algorithm

$m$ CPUs, $k$ GPUs

## Main concept

- Pick any available task $T_i$
- Allocate $T_i$ to CPUs or GPUs
- Schedule it as soon as possible

**Where to allocate an available task $T_i$**

~~If $T_i$ can be executed on GPU before time $\overline{p_i}$:~~

- ~~put $T_i$ on GPU~~

~~Otherwise:~~

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- ~~put $T_i$ on GPU~~

~~Otherwise:~~

- if $\frac{\overline{p_i}}{\underline{p_i}} \leq \sqrt{\frac{m}{k}}$: put it on CPU
- else : put it on GPU

**Theorem**

*QA is $2\sqrt{m/k}+1$ - competitive. This ratio is (almost) tight.*

# What about *easy* cases?

*m* CPUs, *k* GPUs

**Problem with QA**

- Expect the worse: aim at $\Theta(\sqrt{m/k})$-competitiveness
- ☹ Poor performance on *easy* graphs

**Well-known EFT algorithm (Earliest Finish Time)**

- `Terminate each` $T_i$ `as soon as possible;`
- ☺ Greedy version, works great on non-pathological cases
- ☹ Can be really bad: $\geq (\frac{m}{k} + 2)$ OPT

**Can we have both benefits? MixEFT**

- `Run EFT and simulate QA;`
  `When EFT is` $\lambda$ `times worse than QA: switch to QA;`
- Tunable: $\lambda = 0 \rightarrow$ QA ; $\lambda = \infty \rightarrow$ EFT
- $(\lambda+1)(2\sqrt{m/k}+1)$-competitive — conjectured $\max(\lambda, 2\sqrt{m/k}+1)$
- Same idea as ER-LS but pushed to the extreme

# Simulations

$m$ CPUs, $k$ GPUs

**Heuristics (makespan normalized by offline HEFT's)**

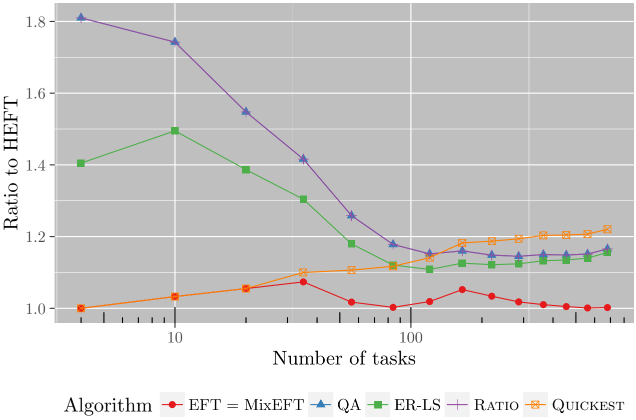
- EFT (= MixEFT as EFT better than QA here)
- QA (switch at $\sqrt{m/k}$)
- ER-LS (= QA + greedy rule: slightly more tasks on GPUs)
- Quickest (= QA with switch at 1: more tasks on GPUs)
- Ratio (= QA with switch at $m/k$: more tasks on CPUs)

**Datasets for $m = 20$ CPUs and $k = 2$ GPUs**

| | |
|---|---|
| Cholesky | 4 types of tasks |
| Synthetic | STG set, 300 tasks, random GPU acceleration ($\mu = \sigma = 15$) |
| Ad-hoc | one chain & independent tasks |

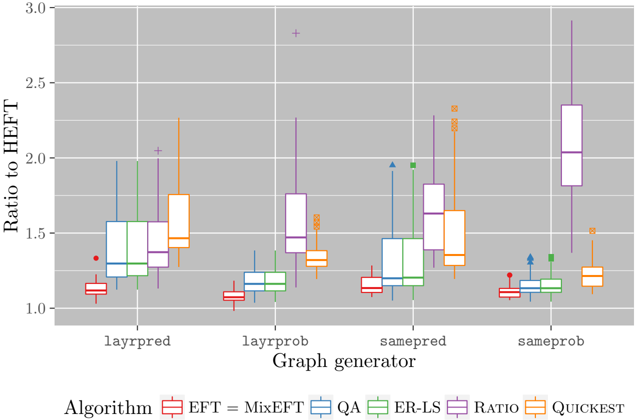
# Results for Cholesky graphs (lower is better)

$m$ CPUs, $k$ GPUs

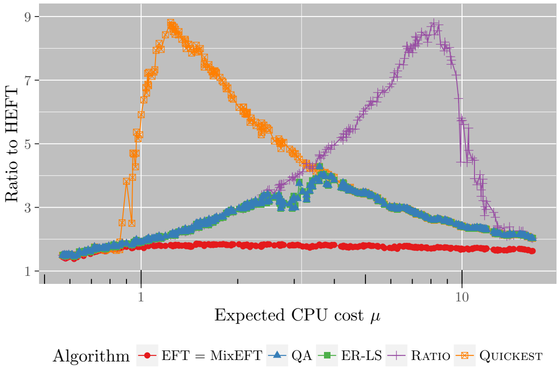


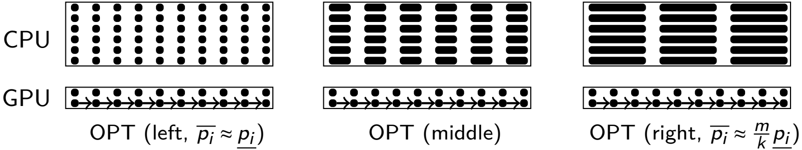

$$\frac{m}{k} = 10 \qquad \sqrt{\frac{m}{k}} \approx 3.3 \qquad \frac{\text{CPU time}}{\text{GPU time}} \in \{28, 26, 11, \underbrace{2}_{\text{POTRF}}\}$$

# Results for synthetic graphs (lower is better)



# Results for 300-tasks ad-hoc graphs (lower is better)



# Conclusion

*m* CPUs, *k* GPUs

## Summary

- No online algo. is $<\sqrt{m/k}$ - competitive
  Additional knowledge or power hardly helps
- QA: $(2\sqrt{m/k}+1)$ - competitive
  MixEFT: compromise effectiveness / guarantees
- Extended to multiple types of processors (not in this talk)

## Perspectives

- Low-cost offline algorithm with constant ratio
- Communication times [Yesterday's talk by Alix Munier-Kordon]
- Parallel tasks